Title: Fair representation

Abstract:

A conjecture of Ryser-Brualdi-Stein from the 1970s is that in a Latin square of order $n$ there exists a transversal (permutation submatrix with distinct symbols) of size $n - 1$. This can be generalized to:

Conjecture: Let $(A_1, \ldots, A_m)$ be a partition of the edge set of $K_{n,n}$. There exists then a perfect matching $F$ that represents the partition fairly in a similar sense to that by which a parliament represents fairly the population of voters, namely $|S \cap A_i| \geq |S||A_i|/|V| - 1$ for all $i$, with the “minus 1” needed only for one $i$.

We prove this for $m = 2, 3$.

We also prove that a partition of the vertices of a path on $n$ vertices into $m$ parts can be fairly represented by an independent set of size $\lceil \frac{n-m}{2} \rceil$.

Both proofs are topological - the first uses Sperner’s lemma, and the second the Borsuk Ulam theorem. I will also discuss the connection of the conjecture to other conjectures on the intersection of matroids.