PROBLEM SET 04 - Selection schemes.

1 Selection schemes

NAME SELECTION SCHEMES USED IN THE SOLUTIONS

Exercise 1.1. How many ways are there to distribute 10 indistinguishable balls into 22 distinguishable bins if
a) each bin may contain at most one ball?
b) each bin may contain unlimited number of balls?
What is the answer, when balls are distinguishable?

Exercise 1.2. How many subsets with more than two elements does a set with 100 elements have?

Exercise 1.3. How many permutations of the letters ABCDEFG contain
a) the string BCD?
b) the strings BA and GF?

Exercise 1.4. How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? [Hint: First position the men and then consider possible positions for the women.]

Exercise 1.5. The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain
a) exactly two vowels?
b) at least two vowels?
c) the same number of vowels and consonants?

Exercise 1.6. How many ways are to arrange around a circural table 10 participants of the wedding of 100 people
a) in total?
b) if the groom and the bride sit by the table next to each other?
Two seatings are considered the same when everyone has the same immediate left and immediate right neighbour.

Exercise 1.7. How many solutions are there to the equation \(x_1 + x_2 + x_3 + x_4 + x_5 = 21\), where \(x_i, i = 1, 2, 3, 4, 5\), is a nonnegative integer such that
a) \(x_1 \geq 1\)?
b) \(x_i \geq 2\) for \(i = 1, 2, 3, 4, 5\)?

Exercise 1.8. A professor packs her collection of 40 issues of a mathematics journal in four boxes with 10 issues per box. How many ways can she distribute the journals if
a) each box is numbered, so that they are distinguishable?
b) the boxes are identical, so that they cannot be distinguished?

Exercise 1.9. How many different strings can be made from the letters in MISSISSIPPI, using all the letters?

Exercise 1.10. A bagel shop has onion bagels, poppy seed bagels, egg bagels, salty bagels, pumpernickel bagels, sesame seed bagels, raisin bagels, and plain bagels (bagels of a given type are indistinguishable). How many ways are there to choose
a) 12 bagels?
b) 12 bagels with at least one of each kind?
c) 12 bagels with at least three egg bagels and no more than two salty bagels?

Exercise 1.11. How many positive integers less than 1,000,000 have the sum of their digits equal to 9?

Exercise 1.12. How many ways can \(n\) books be placed on \(k\) distinguishable shelves
a) if the books are indistinguishable copies of the same title?
b) if no two books are the same, and the positions of the books on the shelves do not matter?
c) (*) if no two books are the same, and the positions of the books on the shelves matter?

Exercise 1.13. How many ways are there to deal hands of seven cards to each of five players from a standard deck of 52 cards?

Exercise 1.14. A shelf holds 12 books in a row. How many ways are there to choose five books so that no two adjacent books are chosen? [Hint: Represent the books that are chosen by bars and the books not chosen by stars. Count the number of sequences of five bars and seven stars so that no two bars are adjacent.]

Exercise 1.15. How many different ways are there to choose 12 donuts from the 21 varieties at a donut shop?
Exercise 1.16. Every day a student randomly chooses a sandwich for lunch from a pile of wrapped sandwiches. If there are six kinds of sandwiches, how many different ways are there for the student to choose sandwiches for the seven days of a week if the order in which the sandwiches are chosen matters?

Exercise 1.17. How many bit strings contain exactly eight 0s and 20 1s if no two 0s stand next to each other?

Exercise 1.18. Thirteen people on a softball team show up for a game.
   a) How many ways are there to choose 10 players to take the field?
   b) How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?
   c) Of the 13 people who show up, three are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?

Exercise 1.19. How many ways are there to choose eight coins from a piggy bank containing 100 identical pennies and 80 identical nickels?

Exercise 1.20. A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes
   a) are there in total?
   b) contain exactly two heads?
   c) contain at most three tails?
   d) contain the same number of heads and tails?

Exercise 1.21. How many bit strings of length 10 contain
   a) exactly four 1s?
   b) at most four 1s?
   c) at least four 1s?
   d) an equal number of 0s and 1s?

Exercise 1.22. A student has three mangos, two papayas, and two kiwi fruits. If the student eats one piece of fruit each day, and only the type of fruit matters, in how many different ways can these fruits be consumed?

Exercise 1.23. A group contains $n$ men and $n$ women. How many ways are there to arrange these people in a row if the men and women alternate?

Exercise 1.24. How many strings of 10 ternary digits (0, 1, or 2) are there that contain exactly two 0s, three 1s, and five 2s?