Additive combinatorics

Winter semester 2016/2017

Series III

3.1. Construct a DCD-sets A such that $E(A) \gg |A|^3$.

3.2. Give an example of a convex set such that $(1_A \circ 1_A)(x) \gg |A|$

3.3.* Show, that for $A \subseteq \mathbb{R}$ we have $|AA + AA| \gg |A|^{3/2-\varepsilon}$.

3.4. Suppose that $A \subseteq \mathbb{F}_p$ and $|A||B| > p$. Prove that $\frac{A-A}{(B-B)\setminus\{0\}} = \mathbb{F}_p$.

3.5.* Suppose that $A, B \subseteq \mathbb{F}_p$ and $|A||B| > 100p$. Prove that $\left|\frac{A-B}{(A-B)\setminus\{0\}}\right| \geqslant p/3$.

3.6. Let $A, B \subseteq \mathbb{F}_p$ be sets with $|A||B| > 100p$. Show that

$$|(A-B)(A-B)| \gg p^{3/4}.$$

3.7. Let $k \in \mathbb{N}$ and let $A = \{a_1, \dots, a_n\}_< \subseteq \mathbb{R}$ be a set such that for all sequences $(a_{i+k} - a_i, \dots, a_{i+1} - a_i)$ dla $i = 1, \dots, n-k$ are distinct. Prove that for each finite set of reals set B we have

$$|A + B| \gg |A||B|^{1/(k+1)}.$$