Additive combinatorics

Winter semester 2016/2017

Series VII

7.1. Let $A \subseteq \mathbb{Z}$ be a Sidon set. Show that there is no 3–element set B such that $B+B \subseteq A$.

7.2.* Let $A \subseteq \mathbb{Z}$ be a finite Sidon set. Prove that for every finite set B one has

$$|A+B| \geqslant \frac{|A|^2|B|}{|A|+|B|-1}.$$

7.3. Let $A \subseteq \mathbb{Z}$ be a Sidon set such that $A-A=\mathbb{Z}_N$. Show that $|\widehat{A}(r)| = \sqrt{|A|-1}$ for every $r \neq 0$.

7.4. Let $A \subseteq \mathbb{Z}$ be a Sidon set $A \subseteq \mathbb{Z}_N$ such that $A-A=\mathbb{Z}_N$. Then $5A=\mathbb{Z}_N$.

7.5. Let $A \subseteq \mathbb{Z}$ be a Sidon set $A \subseteq \mathbb{Z}_N$ such that $A-A=\mathbb{Z}_N$. Prove that if $(S+S)\cap A = \emptyset,\ S \subseteq \mathbb{Z}_N$, then $|S| = O(N^{3/4})$.