Rachunek Prawdopodobieństwa 2

Zestaw zadań nr 2
Termin realizacji: 24 X 2008

1. Consider a symmetric random walk with an absorbing barrier at \( N \) and a reflective barrier at \( 0 \) (so that, when particle is at 0, it moves to 1 at the next step). Find the probability that the particle, having started at \( k \), visits 0 exactly \( j \) times before being absorbed at \( N \). Here \( j \geq 1 \) and \( 0 \leq k \leq N \). (If \( k = 0 \), then the starting point counts as one visit.)

2. \( N + 1 \) plates are laid down around a circular table, and a hot cake is passed between them in the manner of a symmetric random walk: each time it arrives on a plate, it is tossed to one of the two neighboring plates, each possibility having probability \( \frac{1}{2} \). The game stops when the cake has visited every plate at least once. Show that, with the exception of the plate where the cake began, each plate has probability \( \frac{1}{N} \) of being the last plate visited by the cake.

3. Determine \( D_k \) for arbitrary \( p \). (ROZWIĄZANE NA WYKŁADZIE)

4. Determine \( F_k \).

5. Compute \( E|S_n| \). (Here and in all problems below assume \( S_0 = 0 \).)

6. (ZMIANA TREŚCI - zniknęło b) Prove that for \( p = q \) and \( r > 0 \),
   \[ P(M_n \geq r) = P(S_n \geq r) + P(S_n \geq r + 1). \]

7. (ZMIANA TREŚCI - pojawilo się r) Prove that for \( p = q \) and \( r > 0 \),
   \[ P(M_n = r) = \max \{ P(S_n = r), P(S_n = r + 1) \}. \]

8. Let \( T = \min\{n \geq 1 : S_n = 0\} \) and \( p = q \). Compute \( P(T = 2n) \).

9. (Termin: 31 X) Prove that, for \( p = q \),
   \[ P(S_1 \cdots S_{2n} \neq 0) = P(S_{2n} = 0). \]

10. (Termin: 31 X) Let \( p = q \). Compute the probability that the first visit in \( S_{2n} \) takes place at time \( 2k \). (Hint: use the reversal technique)