Rachunek Prawdopodobieństwa 2

Zestaw zadań nr 1

Termin realizacji: 17 X 2008

1. A symmetric random walk takes place on the integers $0, 1, 2, \ldots, N$ with absorbing barriers at 0 and N, starting at k. Show that the probability that the walk is never absorbed is zero.

2. A pack contains m cards, labeled $1, 2, \ldots, m$. The cards are dealt out in a random order, one by one. Given that the label of the kth card dealt is the largest of the first k cards dealt. what is the probability that it is also the largest in the pack?

3. A biased coin is tossed repeatedly. Each time there is a probability p of a head turning up. Let p_n be the probability that an even number of heads has occurred after n tosses (zero is an even number). Show that $p_0 = 1$ and

$$p_n = p(1 - p_{n-1}) + (1 - p)p_{n-1}$$

if $n \geq 1$. Solve this difference equation.

4. A biased coin is tossed repeatedly. Find the probability that there is a run of r heads in a row before there is a run of s tails, where r and s are positive integers.

5. If n couples are seated randomly around a circular table, with men and women alternating, find the probability that nobody sits next to his or her partner.

6. An urn contains a azure balls and c carmine balls, where $ac \neq 0$. Balls are removed at random and discarded until the first time that a ball (B, say) is removed having a different color from its predecessor. The ball B is now replaced and the procedure restarted. This process continues until the last ball is drawn from the urn. Show that this last ball is equally likely to be azure or carmine.

7. 10 percent of the surface of a sphere S is colored blue, the rest is red. Show that, irrespective of the manner in which the colors are distributed (but we must assume that the set of blue points, and thus the set of red points is measurable), it is possible to inscribe a cube in S with all vertices red.

8. The n passengers for a Bell-Air flight in an airplane with n seats have been told their seat numbers. They get on the plane one by one. The first person sits in the wrong seat. Subsequent passengers sit in their assigned seats whenever they find them available, or otherwise in a randomly chosen empty seat. What is the probability that the last passenger finds his seat free?

9. A coin is tossed repeatedly, heads turning up with probability p on each toss. Player A wins the game if m heads appear before n tails have appeared, and player B wins otherwise. Let p_{mn} be the probability that A wins the game. Find p_{mn}.

10. Consider a simple random walk on the set $\{0, 1, 2, \ldots, N\}$ in which each step is to the right with probability p or to the left with probability $q = 1 - p$. Absorbing barriers are placed at 0 and N. Show that the number X of positive steps of the walk before absorption satisfies

$$E(X) = \frac{1}{2}\{D_k - k + N(1 - p_k)\}$$

where D_k is the mean number of steps until absorption and p_k is the probability of absorption at 0.