Logic & Cognition
Reasoning

Mariusz Urbański

Institute of Psychology
Adam Mickiewicz University
Poznań, Poland
http://mu.edu.pl

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Logic and psychology

Macnamara [1986]

Logicians and psychologists often behave like men and women in the orthodox synagogue. Each group knows of the existence of other, but ignoring the fact is considered a proper form of behavior.

What is the relation between logic and psychology? – two (historically) most popular answers:

1. none;
2. the first is a branch of the second.

Logic and cognition: two faces of psychologism

1. How did all this happen – a history of close relationship ending with divorce.
2. Psychologism: the ugly face *vs* the alluring one.
3. Logic and cognitive science in the interaction: interpretations of Wason selection task.
4. An educational concern.

History of close relationship

Logic emerged in Antiquity as an investigation of reasoning, both from the perspective of case-based analysis of their rationality and from the perspective of their structures (w.r.t. ways of organizing scientific knowledge).

So conceived, for many centuries logic stood in a close and natural relationship to the science of actual cognitive processes. As long as both were just parts of philosophy there was no real need for any precise delineations.

Last stage of close relationship

de Morgan [1847, p. 26]

logic is "the branch of inquiry [...] in which the act of the mind in reasoning is considered".

Boole [1854, p. 1]

searched for "the fundamental laws of those operations of the mind by which reasoning is performed [and] some probable intimations concerning the nature and constitution of the human mind".

Mill [1858, p. 7]

logic is the science of "both the processes itself of proceeding from known truths to unknown, and all intellectual operations auxiliary to this".

Last stage of close relationship – psychologism

Beneke [1832, p. 12]

logic is the part of psychology that investigates relations between thinking and the reality.

When Erdmann [1870, vol. 3] coined the term "psychologism" to describe Beneke's views it was merely a neutral description.

Reasons for demarcation

1. Defence of logic against metaphysical designs of Hegel's dialectic method.
2. Emancipation of the psychology, the study of the mind.

A divorce – the ugly face of psychologism

Frege [1884] and Husserl [1900–1901]:

1. Laws of logic are known *a priori*, they are not generalizations of experiences.
2. Laws of logic refer to ideal objects, not to psychological entities.
3. Laws of logic do not govern actual thinking.

Logic (and mathematics) is the most exact of all sciences, while psychology is imprecise and vague. The one has nothing to do with the other.

A dialogue

L: Logic is not a science of reasoning. At most, one of the objects of its interests may be correctness of thinking, related to criteria based on objective – and therefore non-psychological – properties of linguistic expressions, such as their structure, or logical values.

P: Of course, that logic is not a science of reasoning. What is more, it is completely useless in analysis of their correctness, since objective criteria set forth by L are completely unrealistic and do not fit to how people really think.

Reintroducing realistic stance

- formal semantics
- Gödel's limitation theorems
- philosophical logic
- logical pragmatics
- logic in AI
- logical analysis of fallacies

"Practical turn" in logic (Gabbay & Woods [2001])

1. Logic aquires a new task of "systematically keeping track of changing representations of information" (van Benthem [2008, p. 73]).
2. New fields open for logic, with new representations, including those which are not linguistic.

Logic becomes capable of modeling actual cognitive activity of real life agents.

Cognitivism, or New Psychologism: the alluring face

The New Psychologism:

1. claims that it is not the case that logic has nothing to do with how we think, and
2. contests the claim that distinction between descriptive and normative account on analysis of reasoning is disjoint and exhaustive.

An instructive example: Wason selection task (Stenning & van Lambalgen [2008])

Below is depicted a set of four cards, of which you can see only the exposed face and not the hidden back. On each card, there is a number on one of its sides and a letter on the other.

Also below there is a rule wich applies only to the four cards. Your task is to decide which if any of these four cards you must turn in order to decide if the rule is true. Don't turn unnecessary cards. Thick the cards you want to turn.

Rule: *If* (p) *there is a vowel on one side, then* (q) *there is an even number on the other side.*

Cards: | A | | K | | 4 | | 7 |

Typical proportion of choices:

p	p, q	$p, \neg q$	$p, q, \neg q$	misc.
35%	45%	5%	7%	8%

Wason selection task – interpretation of results

Semantic understanding of the task:

1. Truth-functional interpretation of the conditional rule
   1. by a logician
   2. by a psychologist/cognitive scientist
2. The rule as a biconditional
3. Descriptive *vs* deontic interpretation of the rule
4. The rule as a conditional which allows exceptions

"understanding interpretation sometimes leads to clarification of what subjects are trying to do, and that often turns out to be quite different than the experimenter assumes" (Stenning & van Lambalgen [2008, p. 90])

The price

1. two-valuedness: Truth and Falsehood, and nothing else;
2. extensionality: truth-functionality of connectives;
3. monotonicity: irrevocability of entailment;
4. finitistic methods (?).

Some prospects (Urbański [2011])

1. New fields: application of logic to the analysis of actual human reasoning and extraction of logic from them.
2. New methods and tools: because of non-verbal representations and because of factors that are beyond reach of typical logical formalisms (like ordering premises in certain reasoning according to some pragmatic criteria of preference).
3. Neural basis: is logical inference embedded in natural language?
4. An educational concern.

An educational concern

- Logic is a highly incremental discipline for beginners.
- To make sense of the interplay of logic and cognitive science one needs a substantial competence in logic.
- In teaching logic, how to avoid Scylla of trivial narrative handwaving and Charybdis of excessively hermetic formalism?

Is it really necessary for a logician to pay attention to what psychologists and cognitive scientists do (and *vice versa*)?

van Benthem [2008]

One could easily imagine a world where a logician who has created a new logical system does two things instead of one: like now, submit to a logic conference, usually far abroad, but also: telephone the psychologist next door to see if some new nice experiment can be done.

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Reasoning

1. What is reasoning?
2. What are components of reasoning: premises and conclusion?
3. What properties of reasoning and its components are interesting?

Rules governing reasoning (Stanovich [1999], Stenning and van Lambalgen [2008])

Normative rules: reasoning as it should be, ideally.

Descriptive rules: reasoning as it is actually practiced.

Prescriptive rules: these are norms that result from taking into account our bounded rationality, i.e., computational limitations and storage limitations.

and, *a fortiori*, perspectives in research on reasoning.

What is reasoning – who cares?

1. (cognitive) psychologist – from descriptive and prescriptive perspectives;
2. logician – from normative and (possibly) prescriptive perspectives;
3. argumentation theorist – from descriptive and prescriptive perspectives;
4. cognitive scientist – from all three perspectives.

What is reasoning?

Nęcka [2006]

Reasoning is a process of formulating conclusion on the basis of premises, on the basis of previously acquired or commonly available knowledge.

Ajdukiewicz [1974]

Inference is a mental process by which, on the strength of a more or less categorical acceptance of premises, we arrive at the acceptance of the conclusion which we previously either did not accpet at all or accepted less categorically, the degree of certainty of acceptance of the conclusion being not higher than the degree of certainty of acceptance of the premises.

According to Ajdukiewicz reasoning includes:

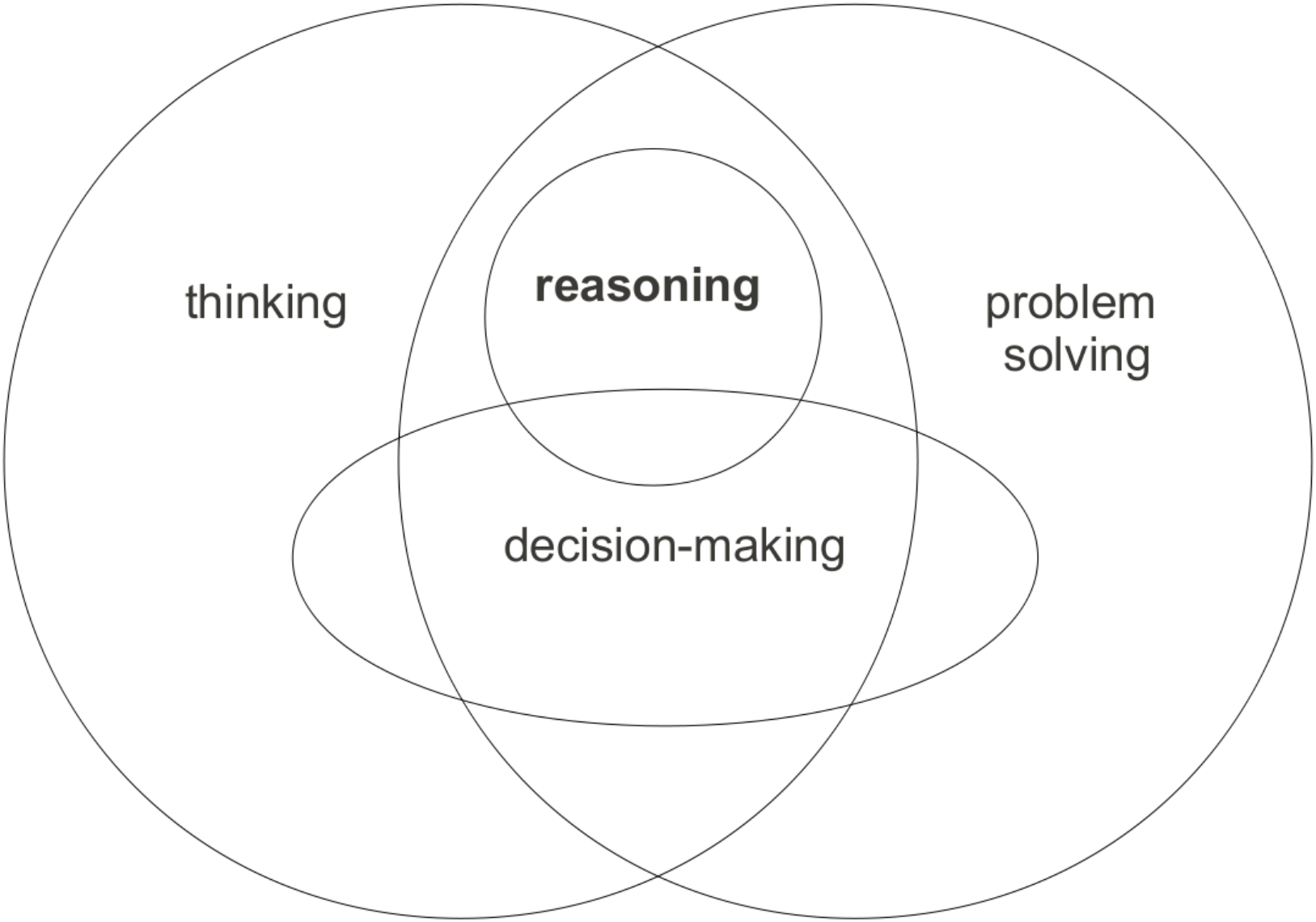
1. (actualized) inferences;
2. potential inferences (like when answering a question "what would happen if something were so and so");
3. processes that subsume the above.

What is reasoning?

Kurtz, Gentner, Gunn [1999]

Human reasoning can be broadly described as a set of cognitive processes by which people take an initial set of information and generate inferences that extend beyond the original data. In this sense, the expectations, generalizations, and assertions people reach in interpreting events and situations can all be considered the result of reasoning. The inferences people produce can range from conclusions justified by formal procedures to sketchy hunches supported by varying degrees of evidence. Reasoning ranges from the really „hard" thinking it takes to formulate an answer to a difficult question or resolve a complex situation to the nearly automatic inferences and predictions that occur in the planning and execution of everyday activities. Reasoning is multifaceted, ubiquitous, and fundamental to human cognition.

So what we are talking about? (Nęcka [2006, p. 443])



Premises and conclusion

- Sentences or propositions? (from Aristotle on)
- Verbal representations or their meanings? (because of, e. g., Grice [1989] and Wiśniewski [1995])
- Multimodal representations? (verbal, sensory, emotional – Thagard [1995], Magnani [2009])
- Neural structures? (Thagard [2007])

Interlude: some virtues of a neural model of reasoning

- We need not assume that there are infinitely many components of reasoning and that some of them will never be the one conceived or spoken – so we need not examine abstract objects whose way of existence is quite mysterious.
- We need not to bother themselves with equally mysterious relationship between abstract propositions and reasoning subjects, flesh and blood entities.
- We avoid the reefs and shoals of various philosophical paradoxes: favorite philosophers' mental states (belief, doubt, etc.) should be regarded as a neural structures or processes.
- Components of reasoning can therefore be conceived as non-verbal representations from various perceptual modalities and reasoning – as a holistic process in which information contained in the premises are collected from multiple sources.

Interlude: some problems of a neural model of reasoning

- What groups of neurons are recruited in processes which are contained in reasoning?
- What are the connections between premises and conclusion? (entailment? logical truth or falsehood of olfactory representations?)
- Maybe this model is too narrow? (reasoning is related to specific structures – the brains of a certain structure and not another)

Interlude: what is important?

Turing [1952]

The important thing is to try to draw a line between the properties of a brain, or of a man, that we want to discuss, and those that we don't. To take an extreme case, we are not interested in the fact that the brain has the consistency of cold porridge. We don't want to say "This machine's quite hard, so it isn't a brain, and so it can't think."

Criteria for evaluation of components of reasoning

- truth values (and vicinity)
- consilience
- simplicity
- analogy
- coherence
- causal relations

Justification of reasoning: definitory rules vs. strategic ones (Hintikka [2007])

Definitory rules

Such rules are merely permissive. They tell us what moves one may make in given circumstances (for example, what are the legitimate moves of chess), but they do not tell anything about which moves are good, bad, or indifferent.

Strategic rules

They tell what to do in order to play well, to increase one's chances of reaching the goal. From the general theory of games, we know that such rules cannot be formulated in move-by-move terms – for instance, in terms of the relationship of premises to a conclusion – but only in terms of complete strategies. (A game theorist would express this point by saying that in general, utilities can absolutely speaking be associated only with entire strategies, not with particular moves.)

Justification of reasoning: definitory rules vs. strategic ones (Hintikka [2007])

Correctness of reasoning assessed from the perspective determined by definitory rules (logical correctness) and the correctness assessed from the perspective of strategic rules (argument strength) need not therefore be the same:

Hintikka, quoting Peirce:

Only in the case of deduction there is no difference between a logically correct argument and a strong one.

As a result, alethic status of premises and conclusion (whatever they are) is quite a different thing than their epistemic status.

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Reasoning: classifications and typologies

1. The structure of reasoning.
2. Simple world: tradition.
3. More complicated world: Peirce.
4. Well-ordered world: Ajdukiewicz.
5. Well(although in a different way)-ordered world: Josephson.
6. Alternative world: Rips.
7. Even more alternative world: Kurtz, Gentner i Gunn.
8. A world (logically) complicated as hell: Stenning i van Lambalgen.

Simple characteristic of the logical structure of reasoning (Stenning and van Lambalgen [2008])

- syntax;
- semantics;
- logical consequence (entailment).

Somewhat abstract characteristic of the logical structure of reasoning (Kurtz et al. [1999])

$$y = F(x, k)$$

- x – the initial available information (premises, more or less);
- F – a summary of the set of computational tools used to manipulate, recombine, or transform the input information
- y – the inferential product of the reasoning process (conclusion, more or less);
- k – stored knowledge and experience.

Somewhat abstract characteristic of the logical structure of reasoning (Kurtz et al. [1999])

$$y = F(x, k)$$

- How the available information x and the inferential product y are represented?
- What are the relations between x and y?
- How the information x is related to computational tools F (and stored knowledge and experience k)?
- How inferences y beyond the available information are generated?

Different answers lead to different notions of reasoning and to different classifications (or typologies) of reasoning.

Simple world (Nęcka [2006, p. 443])

- Deductive reasoning consists in drawing conclusion on the basis of premises, by application of formal rules of logic; logical correctness of the conclusion does not amount to its truth.
- Inductive reasoning consists in introducing new theorems or hypotheses on the basis of finite amount of stored instances.

More complicated world (Peirce [1931–1958])

Besides deduction and induction there is also abductive reasoning. In abduction we try to make sense of some puzzling phenomena.

The surprising fact, C, is observed.
But if A were true, C would be a matter of course.
Hence, there is reason to suspect that A is true.

More complicated world (Peirce [1931–1958])

All the beans from this bag are white. *Deduction*
These beans are from this bag.

These beans are white.

These beans are from this bag. *Induction*
These beans are white.

All the beans from this bag are white.

All the beans from this bag are white. *Abduction*
These beans are white.

These beans are from this bag.

In Peircean syllogistic theory of reasoning each kind of reasoning is characterized by a syllogism of a certain structure.

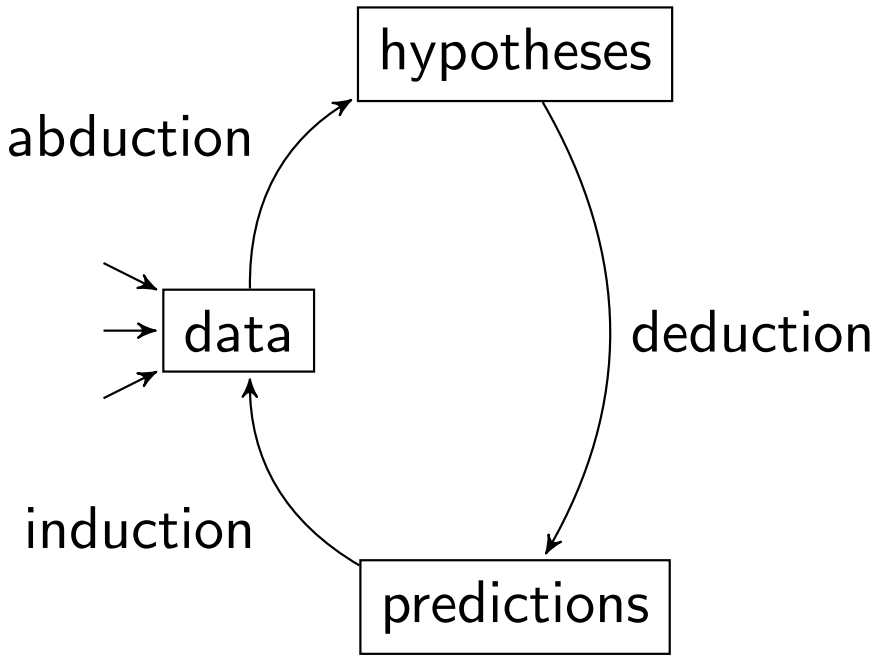
More complicated world (Peirce [1931–1958])

According to his inferential theory abduction, deduction and induction are three stages of one research method. Of these, abduction is the first stage:

> Abduction is the process of forming an explanatory hypothesis. It is the only logical operation which introduces any new idea; for induction does nothing but determine a value, and deduction merely evolves the necessary consequences of a pure hypothesis.

More complicated world (Peirce [1931–1958])

Its [i.e., abduction] only justification is that from its suggestion deduction can draw a prediction which can be tested by induction, and that, if we are ever to learn anything or to understand phenomena at all, it must be by abduction that this is to be brought about.



More complicated world (Peirce [1931–1958])

A man must be downright crazy to deny that science has made many true discoveries. But every single item of scientific theory which stands established today has been due to Abduction.

And this leads to the following distinction:

- *corollarial reasoning*, in which we deductively infer conclusions on the basis of premises;
- *theoremic reasoning*, in which we discover and apply new ideas; they may be rejected in the course of reasoning, but are necessary in order to draw final conclusions.

And to the distinction of definitory and strategic rules.

Well-ordered world (Ajdukiewicz [1974])

Peirce distinguishes types reasoning w.r.t. how inferences y beyond the available information are generated. Classification by Ajdukiewicz is based on (1) complexity of the information processing, and on (2) what are the relations between x and y and k:

- Level I:
  - simple reasoning (in which one process of inference is applied in order to solve a problem);
  - compound reasoning (in which more such processes are employed – like in testing of hypotheses).
- Level II (classification of simple reasoning):
  - deductive (more precisely: subjectively certain reasoning), in which premises logically entail conclusion;
  - ampliative (more precisely, subjectively uncertain reasoning, like reductive, statistic, by analogy etc.), in which – although they are fallible – we can rationally expect that true premises yield highly probable conclusion;
  - logically worthless.

Well(although in a different way)-ordered world (Josephson [1994])

- Level I (criterion – certainty):
  - deductive reasoning;
  - non-deductive reasoning.
- Level II (criterion – explanatory vs predictive outcome):
  - abductive reasoning;
  - predictive reasoning (deduction included);
  - mixed (in. al. by analogy).

Alternative world (Kurtz et al. [1999])

Rips [1990] presents an alternative general framework in his treatment of human reasoning. He structures his discussion around a contrast between what he terms the **strict** and **loose** views of reasoning.

The strict view calls upon algorithmic processes involving the ordered application of abstract procedures to produce definitive conclusions.

The loose view calls upon specific associations, stored instances, statistical summaries, and heuristics that generate continuous-valued predictions or best guesses.

The strict-loose distinction concerns both the way in which processing occurs and the inferential products that result.

Even more alternative world (Kurtz et al. [1999])

What is the role of stored knowledge and experience?

Weak methods are general strategies that can operate without special knowledge of a domain [and] are valuable because of their generality; they provide a means of operating on novel or knowledge-poor domains.

Strong methods make intensive use of specific or abstract represented knowledge [and] are often superior when the appropriate knowledge is present.

A world (logically) complicated as hell (Stenning and van Lambalgen [2008])

A nurse is indicted for murdering several terminally ill patients, who all died during her shifts. No forensic evidence of foul play is found, but the public prosecutor argues that the nurse must have caused the deaths, because she was the only one present at the time of death. This is an example of "credulous" reasoning: an inference is drawn on the basis of data gathered and plausible causal relationships.

The defense countered the prosecutor's argument with an instance of "skeptical" reasoning, by arguing that the cause of death might as well have been malfunctioning of the morphine pumps, and contacted the manufacturer to see whether morphine pumps had had to be recalled because of malfunctioning – which indeed turned out be the case (although in the end it did not help the defendant).

The move of the defence can be viewed as enlarging the class of models considered, thus getting closer to the standard notion of logical consequence where one considers all models of the premises instead of a restricted class.

Reasoning and interpretation of data (Stenning and van Lambalgen [2008, p. 20])

In the psychology of reasoning literature one commonly finds a picture of reasoning as proceeding according to preestablished logical laws, which can be applied by anybody in any circumstances whatsoever.

In fact, however, logic is very much domain dependent in the sense that the valid schemata depend on the domain in which one reasons, *with what purpose*. We therefore view reasoning as consisting of two stages: first one has to establish the domain about which one reasons and its formal properties (reasoning *to* an interpretation) and only after this initial step has been taken can one's reasoning be guided by formal laws (reasoning *from* an interpretation).

Reasoning and interpretation of data – example 1 (Stenning and van Lambalgen [2008, p. 21])

How many people are there?

Once upon a time there was a butcher, a baker, and a candlestick maker. One fine morning, a body was discovered on the village green, a dagger protruding from its chest. The murderer's footprints were clearly registered in the mud...

Reasoning and interpretation of data – example 1 (Stenning and van Lambalgen [2008, p. 21])

How many people are there?

Some person is a baker. Some person is a butcher. Some person is a candlestickmaker. Some person is a murderer. Some person is a corpse.

Reasoning and interpretation of data – example 2

Diagonal axis of a cylinder cross-section is 5 in and the radius of its base is 1.5 in. The height of the cylinder, resulting from rotation around the longer side of the rectangle is:

[A] 5 in [B] 3 in [C] 4 in [D] 6 in

Applying the right logic: decorum (Stenning and van Lambalgen [2008, p. 41])

- planning (adapting to failures of plans during their execution);
- diagnosis;
- causal reasoning;
- reasoning about mental behavior and states;
- interpreting speaker's intentions underlying discourse.

Each of these domains leads to a logic especially suited to that domain. Reasoners have in general little trouble in selecting the logic appropriate to a domain, although they can't define these logics or even precisely decide the grounds on which the choice is made. But are we all aware that we speak in prose?

Positions on the relationship between reasoning and rationality (Stanovich [1999])

- **Panglossian**: Human reasoning competence and performance is actually normatively correct. What appears to be incorrect reasoning can be explained by such maneuvers as different task construal, a different interpretation of logical terms, etc.
- **Apologist**: Actual human performance follows prescriptive rules, but the latter are in general (and necessarily) subnormal, because of the heavy computational demands of normatively correct reasoning.
- **Meliorist**: Actual human reasoning falls short of prescriptive standards, which are themselves subnormal; there is therefore much room for improvement by suitable education
- **Eliminativist**: Reasoning rarely happens in real life, and mainly in institutional contexts such as schools. By contrast, true rationality is adaptiveness: we have developed "fast and frugal algorithms" which allow us to take quick decisions which are optimal given constraints of time and energy.

Bibliography I

- Adler, J. E., Rips, L. J. [2008]. *Reasoning. Studies of Human Inference and Its Foundations*. Cambridge University Press.
- Ajdukiewicz, K. [1974]. *Pragmatic Logic*. D. Reidel P. C., Dordrecht / PWN, Warsaw.
- Beneke, F. E. [1832]. *Lehrbuch der Logik als Kunstlehre des Denkens*. Ernst Siegfrieg Mittler, Berlin, Posen und Bromberg.
- Benthem, J. van [2008]. Logic and Reasoning: do the facts matter? *Studia Logica*, 88(1):67–84.
- Boole, G. [1854]. *An Investigation of the Laws of Thought*. Walton and Maberly, London.
- Charniak, E., McDermott, D. [1985]. *Introduction to Artificial Intelligence*. Addison-Wesley, Reading, MA.
- Erdmann, J. E. [1870]. *Grundriss der Geschichte der Philosophie*. Verlag von Wilhelm Hertz, Berlin, 2 ed.

Bibliography II

- Frege, G. [1884]. *Die Grundlagen der Arithmetik: eine logisch-mathematische Untersuchung über den Begriff der Zahl*. W. Koebner, Breslau.
- Gabbay, D. M., Woods, J. [2001]. The New Logic. *Logic Journal of the IGPL*, 9(2):141–174.
- Grice, H. P. [1989]. *Studies in the Way of Words*. Harvard University Press.
- Hintikka, J. [2007]. Abduction – Inference, Conjecture, or an Answer to a Question? In: *Socratic Epistemology. Explorations of Knowledge-Seeking by Questioning*, p. 38–60. Cambridge University Press.
- Husserl, E. [1900–1901]. *Logische Untersuchungen*. Max Niemeyer Verlag, Halle.
- Josephson, J. R., Josephson, S. G. (red.) [1994]. *Abductive Inference: Computation, Philosophy, Technology*. Cambridge University Press, Cambridge.
- Kurtz, K. J., Gentner, D., Gunn, V. [1999]. Reasoning. W: B. M. Bly, D. E. Rumelhart (red.), *Cognitive Science*, Handbook of Perception and Cognition, 145–200. Elsevier, 2 wyd.

Bibliography III

- Macnamara, J. [1986]. *A Border Dispute. The Place of Logic in Psychology*. MIT Press, Cambridge, MA.
- Magnani, L. [2009]. *Abductive Cognition. The Epistemological and Eco-Cognitive Dimensions of Hypothetical Reasoning*, tom 3 z serii *Cognitive Systems Monographs*. Springer.
- Mill, J. S. [1858]. *A System of Logic, Ratiocinative and Inductive*. Harper & Brothers, New York.
- Morgan, A. de [1847]. *Formal Logic, or The Calculus of Inference, Necessary and Probable*. Taylor and Walton, London.
- Nęcka, E., Orzechowski, J., Szymura, B. [2006]. *Psychologia poznawcza*. Wydawnictwo Naukowe PWN i Academica, Warszawa.
- Peckhaus, V. [2006]. Psychologism and the Distinction between Discovery and Justification. In: J. Schickore, F. Steinle (eds.), *Revisiting Discovery and Justification. Historical and philosophical perspectives on the context distinction*, 99–116. Springer, Dordrecht.

Bibliography IV

- Peirce, C. S. (1931–1958). *Collected Works*, Charles Hartshorne, Paul Weiss, Arthur W. Burks (eds.), Harvard University Press, Cambridge, MA.
- Rips, L. J. [1990]. Reasoning. *Annual Review of Psychology*, 41, 85–116.
- Stanovich, K. E. [1999]. *Who Is Rational? Studies of Individual Differences in Reasoning*. Lawrence Erlbaum, Mahwah, NJ.
- Stenning, K., van Lambalgen, M. [2008]. *Human Reasoning and Cognitive Science*. MIT Press, Cambridge, MA.
- Trendelenburg, F. A. [1843]. *Die logische Frage in Hegel's System. Zwei Streitschriften*. Brockhaus, Leipzig.
- Thagard, P. [1995]. *Abductive Reasoning: Logic, Visual Thinking and Coherence*. MIT Press, Cambridge, MA.
- Thagard, P. [2007]. Abductive inference: From philosophical analysis to neural mechanisms. W: A. Feeney, E. Heit (red.), *Inductive reasoning: Cognitive, mathematical, and neuroscientific approaches*, 226–247. Cambridge University Press, Cambridge.

Bibliography V

- Newman, A. H., Turing, A. M., Jefferson, G., Braithwaite, R. B. [1952]. Can automatic calculating machines be said to think? Broadcast discussion transmitted on BBC (14 and 23 Jan. 1952). The Turing Digital Archive (`www.turingarchive.org`), contents of AMT/B/6.
- Urbański M. [2011]. Logic and Cognition: the Faces of Psychologism, *Logic and Logical Philosophy*, No. 20, p. 175–185.
- Van Eemeren, F. H., Grootendorst, R., Henkemans, F. S., Blair, J. A., Johnson, R. H., Krabbe, E. C. W., Plantin, C., Walton, D. N., Willard, C. A., Woods, J., Zarefsky, D. F. [1996]. *Fundamentals of Argumentation Theory: A Handbook of Historical Backgrounds and Contemporary Developments*. Lawrence Erlbaum Associates, Publishers, Mahwah, NJ.
- Wiśniewski, A. [1995]. *The Posing of Questions: Logical Foundations of Erotetic Inferences*. Kluwer Academic Publishers, Dordrecht–Boston–London.