Multiuser quantum communication networks

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We study a quantum state transfer between spins interacting with an arbitrary network of spins coupled by uniform *XX* interactions. It is shown that in such a system under fairly general conditions, we can expect a nearly perfect transfer of states. Then we analyze a generalization of this model to the case of many network users, where the sender can choose which party he wants to communicate with by appropriately tuning his local magnetic field. We also remark that a similar idea can be used to create an entanglement between several spins coupled to the network.

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I. INTRODUCTION

The fact that certain spin systems can be used for transferring quantum states from a source to a destination spin has been recently observed and studied by a number of authors (see Refs. [1–23]). It has been shown that a perfect transfer of an arbitrary qubit is possible in spin chains [3–9] and some other cases [10–17]. As observed by Bose *et al.* [1], such a system can also be used for creating an entanglement between the source and destination spins. In this paper we develop ideas presented in Ref. [6] and prove that one can get a high fidelity transfer between two spins weakly coupled to an arbitrary connected network provided only that they are placed in a local magnetic field we can control. We also propose a generalization of this scheme and show that a spin network can be used for communication with many destination spins, if a source spin tunes his local magnetic field to the frequency of an appropriate receiver.

The structure of the paper goes as follows. First we describe a model of weakly coupled spins (WCS) and compute the effective Hamiltonian of such systems. Then we use our approach to study communication between spins coupled to a spin chain, a cycle, and, finally, to an arbitrary spin network. In the next section of the paper, we introduce a multi-WCS communication protocol. In this model, a source spin selects an appropriate communication frequency, using which it can effectively communicate with another user of the network. Finally, we use this scheme to create entangled states between network users. In particular, we discuss how to get an almost perfect Bell state for two spins and generalized *W* states in the case of several network users.

II. WEAKLY COUPLED SPINS MODEL

Let us consider two spins coupled to a network G of N spins. We write the Hamiltonian of a network G as

$$H_G = \sum_{(i,j)} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y),$$

where σ_j^k is the kth Pauli matrix corresponding to spin j and we sum over all network edges (i,j). A source spin s is coupled to a spin n_s from G, and a destination spin d is coupled to a n_d from G. The strength of these couplings we denote by $\varepsilon\xi_s$, $\varepsilon\xi_d$, respectively, where we assume that ξ_s, ξ_d are constants independent of N but ε decreases with N. Moreover, spins s and d are placed in local magnetic fields ω_s and ω_d, respectively. Then, the Hamiltonian of the whole system can be written as $H = H_G + H_{sd}$, where H_G is given above and

$$H_{sd} = \varepsilon\xi_s(\sigma_s^x \sigma_{n_s}^x + \sigma_s^y \sigma_{n_s}^y) + \varepsilon\xi_d(\sigma_d^x \sigma_{n_d}^x + \sigma_d^y \sigma_{n_d}^y) + \frac{\omega_s}{2}(\sigma_s^z + 1) + \frac{\omega_d}{2}(\sigma_d^z + 1).$$

We start with the spin s in an arbitrary state $\alpha|0\rangle + \beta|1\rangle$ and the rest of the network in a ground state, i.e.,

$$|\Psi_{t=0}\rangle = \alpha|\underbrace{00}_{sd}\overbrace{0\cdots0}^{\text{network}}\rangle + \beta|\underbrace{10}_{sd}\overbrace{0\cdots0}^{\text{network}}\rangle.$$

Our goal is to transfer the state of spin s to the spin d, i.e., for some time T we wish to have

$$|\Psi_{t=T}\rangle = \alpha|\underbrace{00}_{sd}\overbrace{0\cdots0}^{\text{network}}\rangle + e^{i\varphi}\beta|\underbrace{01}_{sd}\overbrace{0\cdots0}^{\text{network}}\rangle.$$

In the above formula, $e^{i\varphi}$ is an irrelevant phase factor which does not depend on the initial state and could be corrected later by a local operation on spin d. Note that $|00\cdots0\rangle$ is clearly an eigenvector of H, so, in order to transfer a quantum state from s to d it is enough to transfer an excitation between these two spins. Hence we may restrict ourselves to the states which have just one excitation, i.e., we may and shall consider only the evolution of H which takes place in the Hilbert space spanned by vectors $|n\rangle$, where n denotes the position of the excitation and takes values either s, d, or $1,2,\ldots,N$. Therefore, we shall be mostly interested in the case when there is only one excitation in the system and in the evolution $|s\rangle \to e^{i\varphi}|d\rangle$. We remark that the Hamiltonian written in this basis is similar to the network adjacency matrix.

Let $\{\lambda\}$ and $\{|\lambda\rangle\}$ be the sets of the eigenvalues and the eigenvectors of H_G, respectively. Then $H = H_0 + V$, where

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$$H_0 = \omega_s|s\rangle\langle s| + \omega_d|d\rangle\langle d| + \sum_\lambda \lambda|\lambda\rangle\langle\lambda|,$$

$$V = \varepsilon\sum_\lambda (\xi_s g_{s\lambda}|s\rangle\langle\lambda| + \xi_d g_{d\lambda}|d\rangle\langle\lambda| + \text{H.c.}),$$

and

$$g_{s\lambda} = \langle s|\lambda\rangle, \quad g_{d\lambda} = \langle d|\lambda\rangle.$$

Now, let us consider two cases. In the resonant case we have $\omega_s=\omega_d=\lambda'$, where λ' is one of the nondegenerated eigenvalues of G. Then all terms of the Hamiltonian H corresponding to $\lambda\neq\lambda'$ are of the lower order, and the evolution of the system takes place essentially in the subspace L spanned by vectors $|s\rangle$, $|d\rangle$, $|\lambda'\rangle$. The projection of the Hamiltonian onto L in the basis $\{|s\rangle,|d\rangle,|\lambda'\rangle\}$ can be written as

$$H_{\text{eff}}^{s\lambda' d} = \begin{pmatrix} \lambda' & \varepsilon\xi_s g_{s\lambda'} & 0 \\ \varepsilon\xi_s^* g_{s\lambda'}^* & \lambda' & \varepsilon\xi_d g_{d\lambda'} \\ 0 & \varepsilon\xi_d^* g_{d\lambda'}^* & \lambda' \end{pmatrix}.$$

In order to obtain an effective Hamiltonian for the nonresonant case, where $\omega_s\neq\lambda$ and $\omega_d\neq\lambda$ for all λ, i.e., when the local magnetic fields of the source and the destination spins are not tuned to any natural frequency of the network, we follow the perturbation theory approach presented in Ref. [24]. Let us set $H'=e^{iS}He^{-iS}$, where S is a Hermitian operator to be defined later. Note that

$$H'e^{iS}|\lambda\rangle = e^{iS}He^{-iS}e^{iS}|\lambda\rangle = \lambda e^{iS}|\lambda\rangle.$$

Then, expanding e^{iS} and e^{-iS} into power series, we get

$$H' = H_0 + V + i[S,H_0] + i[S,V] + \frac{i^2}{2!}[S,[S,H_0]] + \cdots .$$

Now, let us choose S as

$$S = i\varepsilon\sum_\lambda\left(\frac{\xi_s g_{s\lambda}}{\lambda-\omega_s}|s\rangle\langle\lambda| + \frac{\xi_d g_{d\lambda}}{\lambda-\omega_d}|d\rangle\langle\lambda| + \text{H.c.}\right).$$

Then, in the expression for H' the first order terms in ε vanish, i.e.,

$$V + i[S,H_0] = 0,$$

and $H'=H''+o(\varepsilon^3)$, where

$$H'' = H_0 + i[S,V] + \frac{i^2}{2!}[S,[S,H_0]].$$

The projection of H'' onto the subspace generated by $\{|s\rangle,|d\rangle\}$ can be written as

$$H_{\text{eff}}^{sd} = \begin{pmatrix} \omega_s - A_s & B \\ B^* & \omega_d - A_d \end{pmatrix},$$

where straightforward calculations show that

$$A_s = \varepsilon^2|\xi_s|^2\sum_\lambda \frac{|g_{s\lambda}|^2}{\lambda-\omega_s},$$

$$A_d = \varepsilon^2|\xi_d|^2\sum_\lambda \frac{|g_{d\lambda}|^2}{\lambda-\omega_d},$$

$$B = -\varepsilon^2\xi_s\xi_d^*\left(\sum_\lambda \frac{g_{s\lambda}g_{d\lambda}^*}{\lambda-\omega_s} + \sum_\lambda \frac{g_{s\lambda}g_{d\lambda}^*}{\lambda-\omega_d}\right).$$

Using the above formulas for $H_{\text{eff}}^{s\lambda' d}$ and H_{eff}^{sd}, it is not hard to find sufficient conditions under which the state transfer from s to d occurs. In the resonant case, one should choose all the off-diagonal cases to be of the same absolute value $\beta\varepsilon$. Then, the time of the transfer is, roughly, $\pi/(\sqrt{2}\beta\varepsilon)$. For the nonresonant case, in order to have a nearly perfect transfer between s and d, one should make all the diagonal terms of H_{eff}^{sd} equal, and ensure that the off-diagonal terms do not vanish. Moreover, if the absolute value of the off-diagonal term is $\beta'\varepsilon^2$, then the transfer occurs in time $T\sim\pi/(2\beta'\varepsilon^2)$. It is easy to see that the fidelity of the transfers scales as $F=1-O(g\varepsilon^2)$, where $g=\Sigma_\lambda(g_{s\lambda}^2+g_{d\lambda}^2)$. Thus, under a natural assumption that $g=O(1)$, the fidelity simplifies to $F=1-O(\varepsilon^2)$ (cf. the results presented in Ref. [6]).

Finally, we remark that if we apply the same approach to the case when both ω_d and ω_s are close to a degenerated eigenvalue, we obtain an effective Hamiltonian whose projection $\tilde{H}_{\text{eff}}$ is greater than 3×3. However, typically, no selection of ξ_s and ξ_d can reduce $\tilde{H}_{\text{eff}}$ to a form which guarantees a perfect transfer. Thus, in general, we cannot hope to get a perfect transfer by tuning to a degenerate eigenvalue of H_G.

III. SPIN NETWORKS

In the paper [6], we considered a WCS model for spin chains consisting of N spins, where the source and the destination spins s, d, were coupled to the ends of the chain, and $\omega_s=\omega_d=0$. We showed that such a system admits a high-fidelity transfer of quantum states but the precise description of this phenomenon depends heavily on the parity of N. The reason for that is clear when we compute the effective Hamiltonian for such a system: the case of even N is nonresonant, while for N odd, when zero is an eigenvalue of the chain, a resonant transmission occurs.

Now let us consider the more general case in which s and d are coupled to arbitrary spins of the chain. The eigenvectors and the eigenvalues of the chain are given by

$$|\lambda_k\rangle = \sqrt{\frac{2}{N+1}}\sum_{n=1}^{N} \sin\left(\frac{\pi kn}{N+1}\right)|n\rangle$$

and

$$\lambda_k = 2\cos\left(\frac{\pi k}{N+1}\right),$$

respectively, where N is the chain length and $k=1,2,\ldots,N$. If s and d are coupled to the n_sth and n_dth spin in the chain, respectively, then

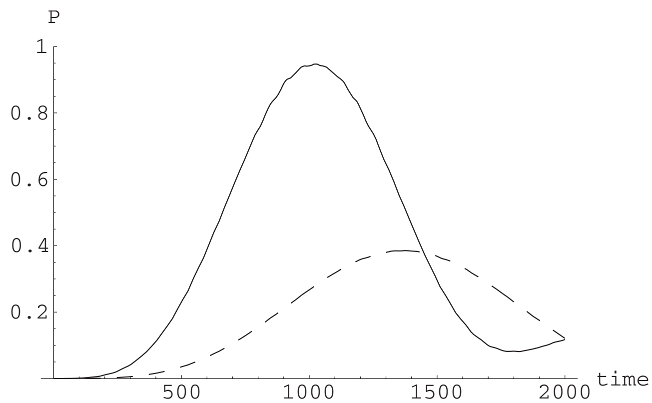

FIG. 1. The probability of the excitation of the destination spin d as a function of time. In this case $N=30$, $n_s=2$, and $n_d=13$. Moreover, we have $\omega_s=\omega_d=\lambda_5$, so the transfer is resonant. The dashed line corresponds to the case $\varepsilon\xi_s=\varepsilon\xi_d=0.01$, and the solid one to the case $\varepsilon\xi_s=0.01$, $\xi_s g_{s\lambda}(2)=\xi_d g_{d\lambda}(13)$.

$$g_{\alpha\lambda}(n_\alpha)=\sqrt{\frac{2}{N+1}}\sin\left(\frac{\pi k n_\alpha}{N+1}\right),$$

for $\alpha=s,d$.

In the symmetric case, in which $n_d=N+1-n_s$ and $\xi_s=\xi_d=\xi$, we have $g_{s\lambda}(n_s)=g_{d\lambda}(N+1-n_s)$ and, as long as $\omega_s=\omega_d$, both the resonant and nonresonant cases give perfect transfer with transfer times of the orders $\sqrt{N}/(\varepsilon\xi)$ and $1/(\varepsilon^2\xi^2)$, respectively.

The asymmetric case, when $n_d\neq N+1-n_s$ needs a bit more attention. If we set $\xi_s=\xi_d$, then $g_{s\lambda}(n_s)\neq g_{d\lambda}(n_d)$ and the transfer occurs with probability bounded from above by a constant smaller than one. However, the transfer can be made perfect by switching to a resonant case. In order to do that, one needs to select the coupling constants ξ_s and ξ_d in such a way that $\xi_s g_{s\lambda}(n_s)=\xi_d g_{d\lambda}(n_d)$, so the condition for a perfect transfer is satisfied (see Fig. 1).

Another example of communicating through a simple spin network is to attach two spins to N-cycle. The eigenvectors and eigenvalues for an N-cycle are

$$|\lambda_k\rangle=\sqrt{\frac{1}{N}}\sum_{n=1}^{N}e^{2\pi ikn/N}|n\rangle$$

and

$$\lambda_k=2\cos\left(\frac{2\pi k}{N}\right),$$

respectively. For a resonant communication through N-cycle, we have only either one or two possible nondegenerated eigenvectors to choose from, namely $|\lambda_N\rangle$ for an odd N, and $|\lambda_{N/2}\rangle$ and $|\lambda_N\rangle$ for an even N. Moreover, one can check that in this case coupling to a degenerated eigenvector cannot lead to a resonant transfer which is nearly perfect. This observation is analogous to a result of Christandl *et al.* [3] which states that the perfect transfer in the chain with equal couplings is possible only for $N=2$ and $N=3$.

Finally, let us consider a general network G with the source spin s and the destination spin d attached to the nodes n_s and n_d, respectively. A possible transfer of quantum states between s and d depends on the eigenvectors $\{|\lambda\rangle\}$ of G. Note, that each localized state of the network $|n\rangle$ is a superposition of eigenvectors $\{|\lambda\rangle\}$. In particular, s and d cannot communicate unless there exists at least one eigenvector $|\lambda'\rangle$ that gives nonzero scalar product with both $|n_s\rangle$ and $|n_d\rangle$; such an eigenvector can be viewed as a communication channel. Note however, that for every connected network in which all coupling constants are real, such a vector exists. Indeed, if λ' is the largest eigenvalue of G then, by Perron-Frobenius theorem, $|\lambda'\rangle$ is a vector which corresponds to a stationary distribution of a particle in a classical random walk on G and so, for connected G, we have $\langle n|\lambda'\rangle>0$, for $n=1,2,\ldots,N$. Observe also that for a connected G, the largest eigenvalue λ' is always nondegenerate, so we can achieve a near perfect transfer setting $\omega_s=\omega_d=\lambda'$, and choosing ξ_s and ξ_d so that $\xi_s g_{s\lambda'}=\xi_d g_{d\lambda'}$. We remark however that for some networks it is not possible to achieve a nearly perfect transfer in a nonresonant way, because for any choice of ξ's and ω's either the diagonal terms of the 2×2 projection of the effective Hamiltonian are not equal, or the off-diagonal terms vanish.

IV. MULTIUSER QUANTUM NETWORK

Let us recall that in order to have a nearly perfect transfer between s and d, we should set $\omega_s=\omega_d$; it is also not hard to check that the fidelity of transfer drops down rapidly when the difference $|\omega_s-\omega_d|$ grows. This observation suggests the following multiuser generalization of our communication protocol. Assume that spins $d_1,d_2,\ldots$ are coupled to some spins of a spin network G and placed in local magnetic fields $\omega_{d_1},\omega_{d_2},\ldots$, respectively. Then another spin s, coupled to a spin from G as well, can communicate with any spin d_k by making its own magnetic field ω_s equal to ω_{d_k}, and calibrating appropriately the coupling strength ξ_s. To guarantee a high fidelity of the state transfer from s to d_k, we should ensure that the distance between $\omega_s=\omega_{d_k}$ and the other frequencies ω_{d_i}, $i\neq k$, as well as between ω_s and the eigenvalues λ of G (except, perhaps one of them, when in the resonant case we have $\lambda'=\omega_s$) is large enough. On the other hand, a large magnetic field ω_k slows down the transfer from s to d_k. Thus, choosing frequencies $\omega_1,\omega_2,\ldots$, we should keep in mind both the fidelity of the transfer (which decreases with the distance between ω_i and the closest eigenvalue) and the time of the transmission (which may increase considerably when ω_i is large, say, much larger than the largest eigenvalue of the network).

We remark that if in such a protocol another user sets his frequency to the communication frequency, the information will get entangled between him and the intended receiver. An example of such a disturbance in a system of three spins coupled to a 21-cycle is presented in Fig. 2. At this point, it is worth mentioning that although resonant transfer seems to

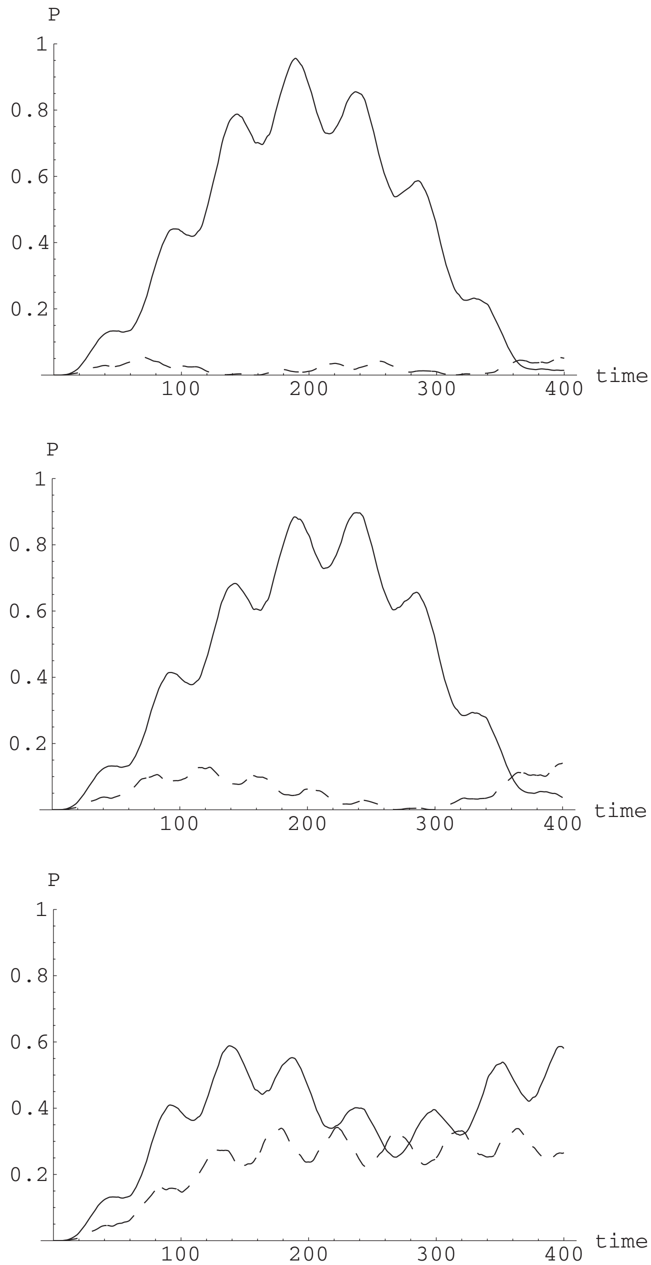

FIG. 2. The probability of the state transfer for a 21-cycle. Three spins s, u, and d, are coupled to the third, tenth, and the eighteenth spins of the cycle, respectively. The coupling strength is $\varepsilon\xi=0.1$, and $\omega_s=\omega_d=-0.9$. The dashed line represents the population of u, the solid line represents that of d. The top figure corresponds to the case when $\omega_u=-0.85$, the middle figure to $\omega_u=-0.87$, and the bottom figure to $\omega_u=-0.89$.

be faster, in the case of degeneration of network's eigenvalues it is necessary to use the off-resonant one if there are more network users than distinct eigenvalues.

V. ENTANGLEMENT GENERATION

The WCS system can also be used for generation of a perfect entanglement. Let us consider first a nonresonant communication between two users. After the time equal to

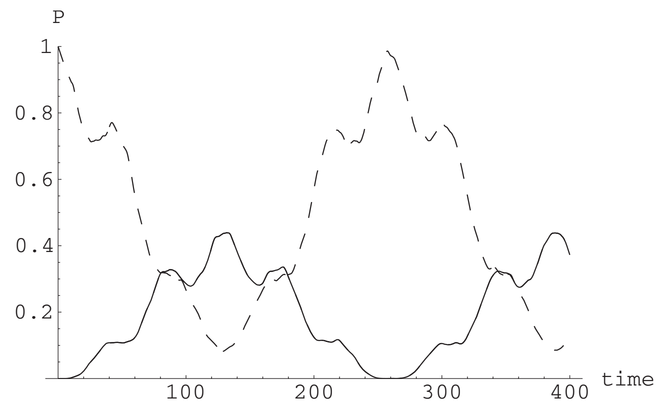

FIG. 3. The probability of the excitation of three spins coupled to a 21-cycle: s_1 coupled to the third, s_2 to the twelfth, and s_3 to the fifteenth spins of the cycle. The dashed line represents the excitation of s_1, the solid line represents that of s_2 and s_3. Here we set $\omega_{s_1}=\omega_{s_2}=\omega_{s_3}=-0.9$; the coupling strength is $\varepsilon\xi=0.1$. The W state is obtained at times when two lines meet.

one-half of the transfer time, the state of the system is $\frac{1}{\sqrt{2}}(|s\rangle+e^{i\phi}|d\rangle)$, where the phase factor ϕ can be found by straightforward calculation. This state can be also written in the form

$$\frac{1}{\sqrt{2}}(|\underbrace{10}_{sd}\overbrace{0\cdots0}^{\text{network}}\rangle+e^{i\phi}|\underbrace{01}_{sd}\overbrace{0\cdots0}^{\text{network}}\rangle)$$
$$=\frac{1}{\sqrt{2}}(|01\rangle+e^{i\phi}|10\rangle)_{sd}|0\cdots0\rangle_{\text{network}},$$

which clearly corresponds to a maximum entanglement between s and d.

In order to obtain W state, one must consider three spins and a nonresonant effective Hamiltonian of the form

$$H_{\text{eff}}^{\text{multi}}\approx\begin{pmatrix}\gamma & \alpha & e^{i\varphi}\alpha\\ \alpha^* & \gamma & \beta\\ e^{-i\varphi}\alpha^* & \beta^* & \gamma\end{pmatrix}.$$

Let us set the initial conditions as $|\Psi(0)\rangle=|s_1\rangle$. After an easily computable time, the state of the system becomes a W state $\frac{1}{\sqrt{3}}(|s_1\rangle+e^{i\phi}|s_2\rangle+e^{i\theta}|s_3\rangle)$, for some ϕ and θ (cf. Fig. 3).

We can also use a resonant transfer to get an entanglement for $m\geqslant2$ users. We consider two cases of W state generation: between all users and between all users except the sender. The second one seems more advantageous because we use the same Hamiltonian all the time, therefore we do not disturb the system during the process.

In the first case we apply the following procedure. First, one user s_1 in an initial state $|\Psi(0)\rangle=|s_1\rangle$ couples to a nondegenerated eigenvalue λ' of the network and waits until the system evolves into $|\lambda'\rangle$. This evolution is described by the following effective Hamiltonian:

$$H_{\text{eff}}^1=\lambda'|\lambda'\rangle\langle\lambda'|+\lambda'|s_1\rangle\langle s_1|+(\varepsilon_{\xi_1}g_{s_1\lambda'}|s_1\rangle\langle\lambda'|+\text{H.c.}).$$

Then all m users couple to excited $|\lambda'\rangle$ state via the effective Hamiltonian

$$H_{\text{eff}}^2 = \lambda'|\lambda'\rangle\langle\lambda'| + \sum_i \lambda'|s_i\rangle\langle s_i| + \left(\varepsilon\sum_i^M \xi_i g_{s_i\lambda'}|s_i\rangle\langle\lambda'| + \text{H.c.}\right),$$

where $\xi_i g_{s_i\lambda'} = \xi_j g_{s_j\lambda'}$ for all pairs $\{i,j\}$, and wait until the system evolves to the state

$$\frac{1}{\sqrt{M}}\sum_{j=1}^{M} e^{i\varphi_j}|s_j\rangle.$$

Note that this method is similar to the dynamics of the spin star network presented in Ref. [23]. Observe also that one can get rid of all relative phases by local one qubit operations.

In the second case, every user but the sender s_0 couples equally strong to the network eigenvalue λ'. Note that for the initial state $|\Psi(0)\rangle = |s_0\rangle$ the evolution given by the effective Hamiltonian takes place only in three-dimensional Hilbert space spanned by the vectors $|s_0\rangle$, $|\lambda'\rangle$, and $|W\rangle$, where $|W\rangle = \frac{1}{\sqrt{m}}\sum_{i=1}^{m}|s_i\rangle$ is the W state between m users. Because of this fact, one can learn the evolution of the system studying a behavior of a chain of length 3 (see Ref. [25]). Note, however, that in this case, in order to get equal chain couplings, the sender must couple $\sqrt{m}$ times stronger than the other users of the network.

VI. SUMMARY

In the paper, we generalized a number of earlier results on quantum information transmission between a source spin and a destination spin using a simple spin network. We show that a near-perfect state transfer between two spins is possible through a large class of networks, provided that we can control local magnetic field in which the source and the destination spins are embedded. We also point out that a source spin can choose the destination spin from a number of users of the network by an appropriate tuning, i.e., by carefully selecting its local magnetic field. The very same mechanism can be used to generate a multispin entanglement state.

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