Comment on "Scheme for teleportation of an unknown atomic state without the Bell-state measurement"

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Recently, Ye and Guo [Phys. Rev. A **70**, 054303 (2004)] have presented a scheme for implementing quantum teleportation of atomic states in cavity QED. In this Comment, we show that contrary to the authors' claim, the scheme is based on Bell-state measurement.

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Recently, Ye and Guo [1] have presented a scheme for implementing quantum teleportation [2] of an unknown atomic state in cavity QED. In this scheme, simultaneous interaction of two appropriately prepared identical two-level atoms (atom 2 and atom 3) with a suitably detuned single-mode cavity field produces a maximally entangled state of these atoms. Next, atom 1 in an unknown state (to be teleported) and one of the atoms (atom 2) from the maximally entangled pair again simultaneously interact with the same cavity field for an appropriate time after which a measurement in their energy eigenbasis is performed. This allows teleportation with a success rate of 50% as for two out of the four measurement results the unknown state of atom 1 is exactly recovered on atom 3 while for the remaining results the information about the unknown state is irreversibly lost.

However, the authors claim in the title as well as throughout their paper that they implement teleportation without the Bell-state measurement (BSM). Teleportation without BSM is, of course, not possible since only information about BSM transmitted from a sender to a receiver allows a faithful recovery of an unknown state. Indeed, in this scheme a BSM, albeit an incomplete one, is performed as shown below. Incomplete information about the BSM reduces the probability of success of the scheme. This situation is analogous to BSM with linear optical elements where only two out of the four Bell states can be distinguished [3]. In the present case, again, only two of the maximally entangled states can be distinguished giving a 50% chance of successful teleportation.

Following Ref. [1] let atom 1 be in the unknown state $|\psi\rangle_1=\alpha|g\rangle_1+\beta|e\rangle_1$, where $|g(e)\rangle$ are the ground (excited) states of the atom. A maximally entangled state $|\eta\rangle_{23}=(1/\sqrt{2})(|eg\rangle-i|ge\rangle)$ of atom 2 and 3 serves as the channel for teleportation. The joint state of the three atoms is given by

$$|\psi\rangle_1|\eta^-\rangle_{23}=\frac{1}{2}(-i|\eta^+\rangle_{12}|\psi\rangle_3+i|\eta^-\rangle_{12}\sigma_z|\psi\rangle_3 - i|\chi^+\rangle_{12}\sigma_x|\psi\rangle_3-|\chi^-\rangle_{12}\sigma_y|\psi\rangle_3, \quad (1)$$

where

$$|\chi^\pm\rangle=\frac{1}{\sqrt{2}}(|gg\rangle\pm i|ee\rangle), \quad |\eta^\pm\rangle=\frac{1}{\sqrt{2}}(|eg\rangle\pm i|ge\rangle) \quad (2)$$

is the natural maximally entangled basis (MEB) corresponding to this setup. Notice that the MEB is locally equivalent to the standard Bell basis. A measurement of atoms 1 and 2 in this MEB (which is a BSM) would in principle allow successful teleportation.

In most physical implementations BSM is performed as follows [4]. First, the MEB is unitarily transformed into some easily measurable product basis. A measurement in this product basis then uniquely reveals the result of the BSM. For the case at hand, a unitary transformation of the form $U=(A\otimes\mathbb{1})$ CNOT, where CNOT denotes controlled-NOT of atoms 1 and 2, where

$$A=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ -i & 1\end{pmatrix} \quad (3)$$

would convert the given MEB into the product energy eigenbasis of the two atoms $\{|ee\rangle_{12},|eg\rangle_{12},|ge\rangle_{12},|gg\rangle_{12}\}$. The two bits of information obtained from the measurement in this product basis then completely determines the BSM and when sent from a sender to a receiver allows faithful teleportation.

The scheme of Ye and Guo essentially follows the same procedure. However, the unitary transformation $U=e^{-i(\pi/4\lambda)H}$ generated by the Hamiltonian

$$H=\lambda(|e_i\rangle\langle e_i|+|e_j\rangle\langle e_j|+\sigma_i^+\sigma_j^-+\sigma_j^+\sigma_i^-) \quad (4)$$

is used, which only partly disentangles the MEB

$$|\eta^-\rangle_{12}\rightarrow e^{-i(3\pi/4)}|ge\rangle_{12}, \quad |\eta^+\rangle_{12}\rightarrow e^{-i(\pi/4)}|eg\rangle_{12}, \quad (5)$$

$$|\chi^\pm\rangle_{12}\rightarrow|\Phi^\pm\rangle_{12}=\frac{1}{\sqrt{2}}(|gg\rangle_{12}\pm|ee\rangle_{12}). \quad (6)$$

A measurement in the energy eigenbasis preceded by this unitary transformation allows the discrimination of two of the maximally entangled states $|\eta^\pm\rangle$. Thus, a partial BSM is indeed performed in this scheme.

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