Discrete Mathematics 2

Problem set #5

Due: Wednesday, January 9

1. Show that for all hypergraphs $\mathcal{F}$ with vertex set $[n]$ and all $1 \le i < j \le n$
   (a) $|\mathcal{S}_{ij}(\mathcal{F})| = |\mathcal{F}|$,
   (b) If $\mathcal{F}$ is k-uniform, then so is $\mathcal{S}_{ij}(\mathcal{F})$,
   (c) If $\mathcal{F}$ is t-intersecting, then so is $\mathcal{S}_{ij}(\mathcal{F})$,
   (d) If $\mathcal{F}$ has no matching of size s, then the same is true for $\mathcal{S}_{ij}(\mathcal{F})$.
2. Show that if $\nu(\mathcal{F}) = s$ and $|\mathcal{F}| = m^{(k)}(n, s)$, then also $\nu(\mathcal{S}_{ij}(\mathcal{F})) = s$.
3. Show that if $\mathcal{F}$ is a k-graph on vertex set $[n]$ and $1 \le i < j \le n$ then

$$\partial(\mathcal{S}_{ij}(\mathcal{F})) \subset \mathcal{S}_{ij}(\partial\mathcal{F}).$$

4. Prove that after finitely many shift operations $\mathcal{S}_{ij}$ applied to a k-graph $\mathcal{F}$, we will arrive at a shifted (stable) k-graph.
5. For a fixed n, determine the largest s for which

$$\binom{s-1}{2} + (s-1)(n-s+1) \ge \binom{2s-1}{2}.$$

6. Prove that for all $k, s \ge 2$

$$m^{(k)}(sk, s) = \binom{ks-1}{k}.$$

7. Prove that for all k, n and $1 \le s \le n-1$, the maximum number of edges in a (k, k)-graph with n vertices in each partition class and with no matching of size $s+1$, equals sn^{k-1}. Show also that for $n = 2$ there is more than one extremal hypergraph with the above property.