# Discrete Mathematics 2

## Problem set #8

Due: Monday, January 8

1. Using only the bound $R(3) \leq 6$ and the formula $R(2, l_1, \ldots, l_r) = R(l_1, \ldots, l_r)$ show that

   (i) $R(3,5) \leq 14$

   (ii) $R(4) \leq 18$

   (iii) $R(3,3,3) \leq 17$

2. Give as good upper bound as you can for $R(4,5)$.

3. Show that $R(k) = o(4^k)$ as $k \to \infty$

4. Show that $R(3,l) \leq (l^2+3)/2$.

5. Show (by induction) that $2^r < R(3;r) \leq \lfloor r!e \rfloor + 1$. Hint: if you can't do the upper bound, try a bit easier $R(3;r) \leq 3r!$.

6. Prove that for every 2-coloring of the edges of $K_n$ there is a monochromatic copy of $C_n$ or one consisting of just two monoochromatic paths.

7. Using the deletion method (and not the LLL), prove Shearer's bound $R(k) \geq \frac{1}{e}k2^{k/2} - 2^{k/2+1}$. Hint: take a random 2-coloring and consider the random variable $X$ equal to the number of monochromatic cliques $K_k$.

8. Assume that for two integers $n \geq m \geq 2$, we have $(m-1)|(n-1)$. Let $T_m$ be any tree on $m$ vertices and $S_n$ be a star with $n$ rays. Show that $R(T_m, S_n) = n + m - 1$. Hint: for the upper bound use induction.

9. Show that $R(2K_3) = 10$. (Here $2K_3$ stands for a pair of vertex-disjoint triangles.) Generalize the construction to show the lower bound $R(nK_3) \geq 5n$ for all $n \geq 2$.

10. Let $P_3$ be he path with three edges. Give as good lower and upper bounds as you can on $R(P_3;r)$.

11. How many triangles are there in $L(K_5)$, the line graph of $K_5$. Give two solutions: (i) by direct counting and (ii) by considering the complement and applying a corollary from Goodman's bound.

12. Show that $K_6 \to (C_4)$.

13. Let $G = K_8 - C_5$ be the graph obtained from $K_8$ by removing 5 edges forming a copy of $C_5$. Show that (i) $G$ is triangle-free and (ii) $G \to (K_3)$.