Graph Theory II

Problem Set #7

due: Tuesday, May 9

1. Show that the complements of bipartite graphs are perfect. Hint: Use König's Theorem.
2. Show that the complements of odd cycles are not perfect.
3. A graph is called an *interval graph* if there exists a set $\{I_v : v \in V(G)\}$ of real intervals such that $I_u \cap I_v \neq \emptyset$ iff $uv \in E(G)$. Show that every interval graph is chordal.
4. Show that perfectness is not closed under edge deletion.
5. Show that Theorem 12 implies Theorem 14 and that the latter, in turn, implies Theorem 13.
6. Using results on perfect graphs, give a 1-line proof of the dual König Theorem: In every bipartite graph G without isolated vertices, the minimum number of edges meeting all vertices equals $\alpha(G)$.
7. Show that for every line graph H we have $\chi(H) \in \{\omega(H), \omega(H) + 1\}$. Hint: Let $H = L(G)$. Compare $\omega(H)$ with $\Delta(G)$.