Graph Theory II

Problem Set #5

due: Wednesday, April 12

1. Go over the proof of Brooks Theorem and verify that it goes through in the more general setting of colorings from lists.

2. For every $k \geqslant 3$, find a bipartite graph G with $ch(G) \geqslant k$. Hint: Consider a $K_{n,n}$ with list being k-subsets of a k^2-element set.

3. What is the complement of the line graph $L(K_5)$.

4. Not using Thomassen's Theorem show that for every planar graph $ch(G) \leqslant 6$.

5. Determine the chromatic index of the Petersen graph.

6. Determine the chromatic index of every complete graph.

7. Show that every hamiltonian cubic graph is 3-edge-colorable.

8. Let the vertices of maximum degree in a graph G induce a forest. Show that $\chi'(G) = \Delta(G)$. Hint: This seems to be difficult, so first try to solve a simpler version where we assume that the vertices of maximum degree form an independent set; or, still simpler, that there is just one vertex of maximum degree. I suspect that Statement S_k from Schrijver's proof of Vizing's Theorem may come handy.