Graph Theory II

Problem Set #4

due: Wednesday, March 29

1. Draw two different triangulations on 7 vertices.
2. Prove that every plane (bipartite) graph on n vertices has at most $3n-6$ $(2n-4)$ edges. Hint: Apply Euler's formula and double counting.
3. Find a subdivision of K_5 or $K_{3,3}$ in Petersen Graph. Then, give another proof of non-planarity of Petersen Graph based on Euler's formula.
4. Show that $\chi(G) \leqslant 6$ for every planar graph G.
5. Show that every planar graph without C_4 has at most $(15n-30)/7$ edges. Find a graph which achieves this bound. Hint: Observe that no two triangles share an edge.
6. A soccer ball is always a polyhedron consisting of pentagons and hexagons. Show that no matter what the size of the ball is, the number of pentagons is always the same. What is that number? Assume that exactly 3 faces meet at every corner (vertex).
7. Given are two maps: one on Earth, another on the Moon. Every country possesses a connected region on each celestial body. How many colors suffice to properly color all countries on both maps, if both parts of every country have to be colored by the same color. How many colors are necessary? Hint: Don't use the Four Color Theorem as it only gives an upper bound of 16. For the lower bound, consider decomposing a clique into two planar graphs.