Graph Theory II

Problem Set #3

due: Wednesday, March 22

1. Show that every graph G has a vertex ordering for which the greedy algorithm uses only $\chi(G)$ colors.
2. For every n find a bipartite graph on $2n$ vertices ordered in such a way that the greedy algorithm uses rather n than 2 colors.
3. Prove Proposition 2, that is, prove that $\chi(G) \leqslant col(G) = \max\{\delta(H) :\ H \subseteq G\} + 1$.
   Remark: Prove both, the inequality and the equality.
   Hint: For equality, in one direction build the ordering of vertices from the end; in the other, consider the subgraph with largest minimum degree.
4. Find a lower bound on the coloring number $col(G)$ in terms of the *maximum density* $m(G) = \max\{d(H) :\ H \subseteq G\}$, where the *density*

$$d(H) = \frac{||H||}{|H|} = \frac{\sum_{v \in V(H)} deg_H(v)}{2|H|}$$

   is a half of the average vertex degree of H.
   Hint: Consider a subgraph with largest density which has the smallest number of vertices.
5. Show that Theorems 6 and $6'$ are, indeed, equivalent.